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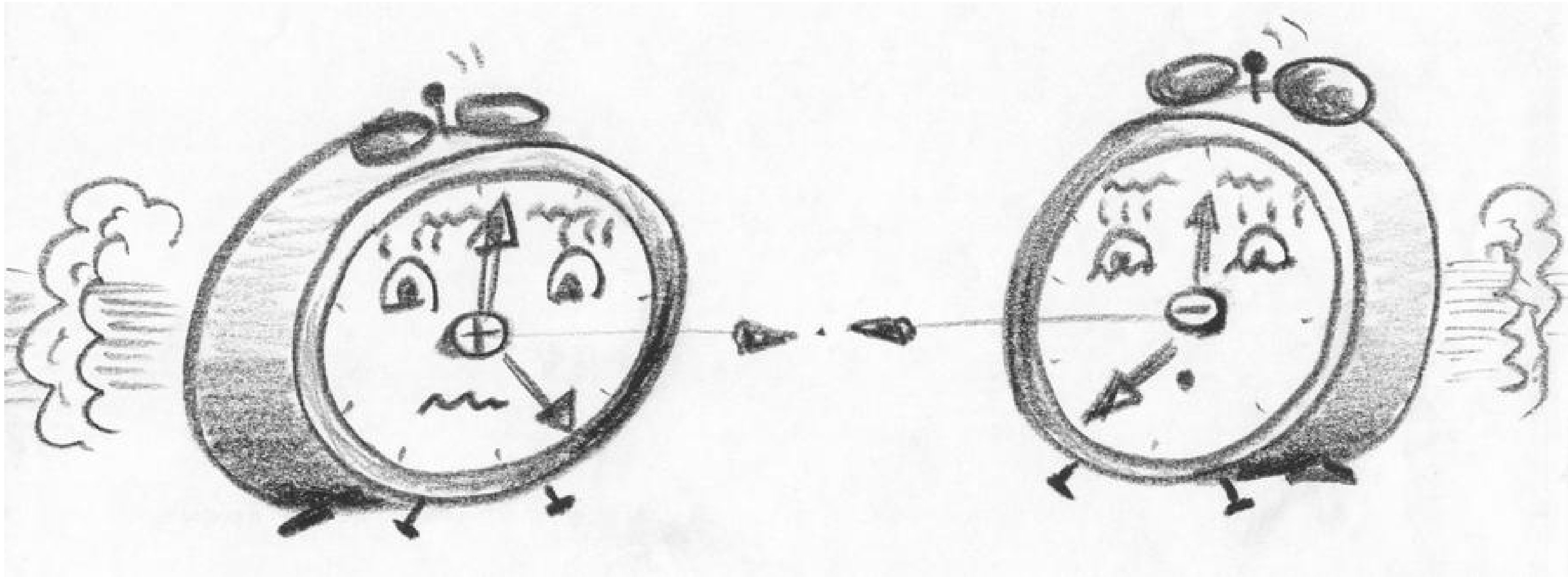
The effects of top quark and W boson finite widths on the measurement of the top quark mass

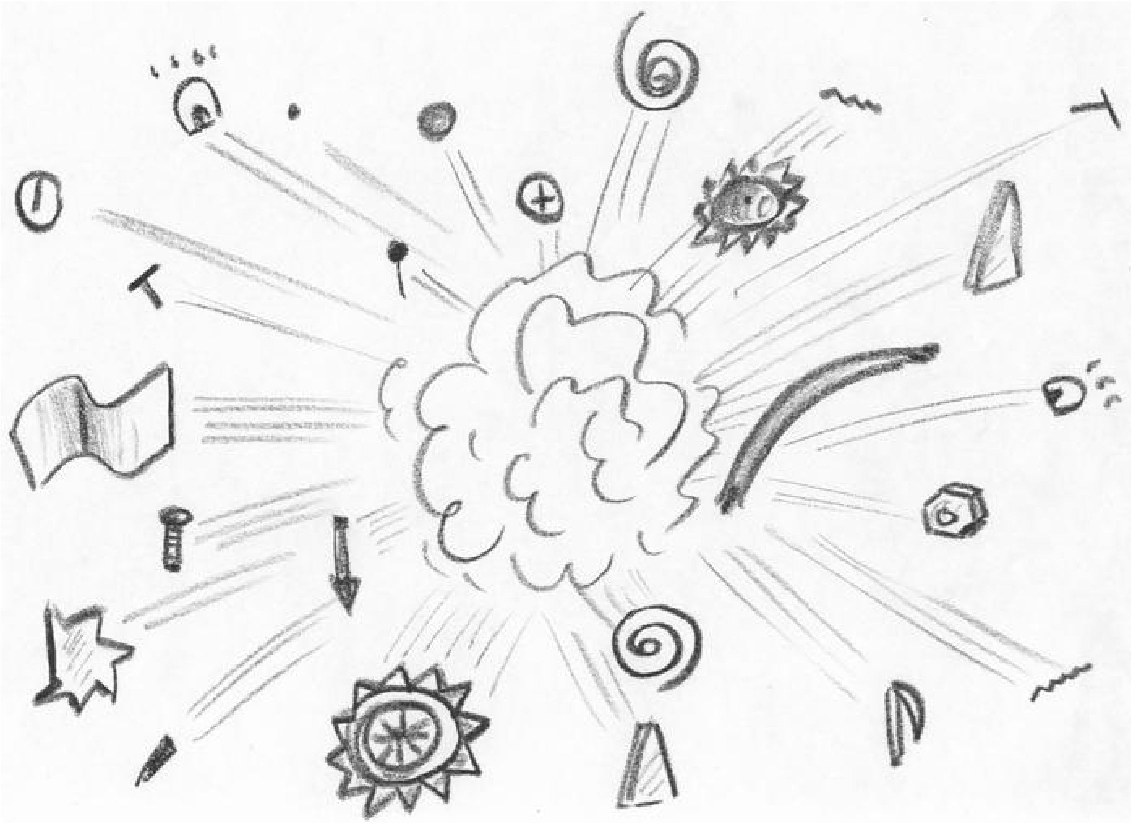
Diploma Thesis

Supervisor: RNDr. Rupert Leitner, DrSc.



"Proton–proton collisions are like smashing two pocket watches together to see how they are put together." R. P. Feynman

...after the collision...



The purpose of my diploma thesis

- Get to know what the title of my thesis means :-)
- Get accustomed with the decay modes of top quark, its production and cross sections
- Model the effect of its decay width Γ_t on the measurement of m_t
- Learn to work with parton distribution functions
- Try to incorporate W width into the phenomenology of $t\bar{t}$ decay
- Get some impression of what can data analysis for $t\bar{t}$ events look like
- ...and of course work with PAW, Fortran, Root, LaTeX , Maple ... and don't run mad :-)

Basic top quark properties

- The heaviest particle both among bosons and fermions (so far...)
- The top quark pole mass: $(174.3 \pm 5.1)\,\mathrm{GeV}$
- The full decay width corresponding to this mass: $1.4\,\mathrm{GeV}$
- Spin and parity J^P (SM prediction): $\frac{1}{2}^+$
- Weak isospin projection eigenvalue T_3: $+1/2$
- Charge Q: $+2/3|e|$
- Top (Truth) T: $+1$
- Perhaps the question isn't why it is so heavy, but why other leptons are so light!

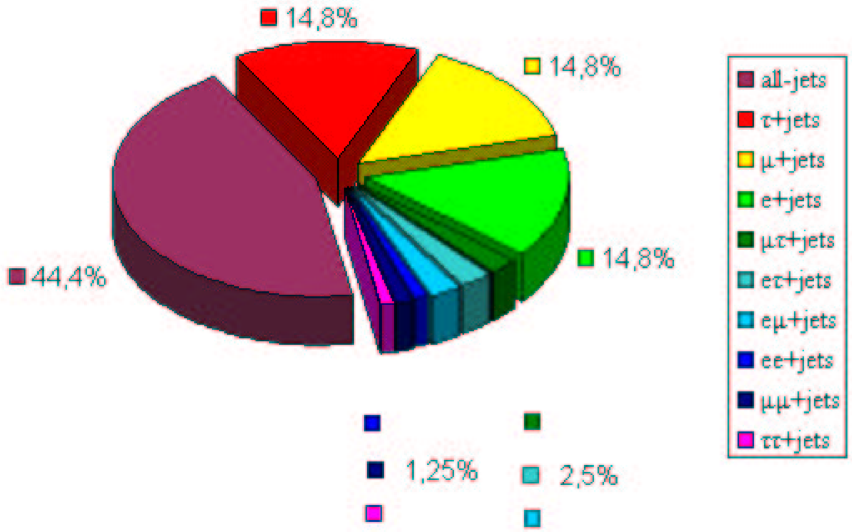
Top decay modes

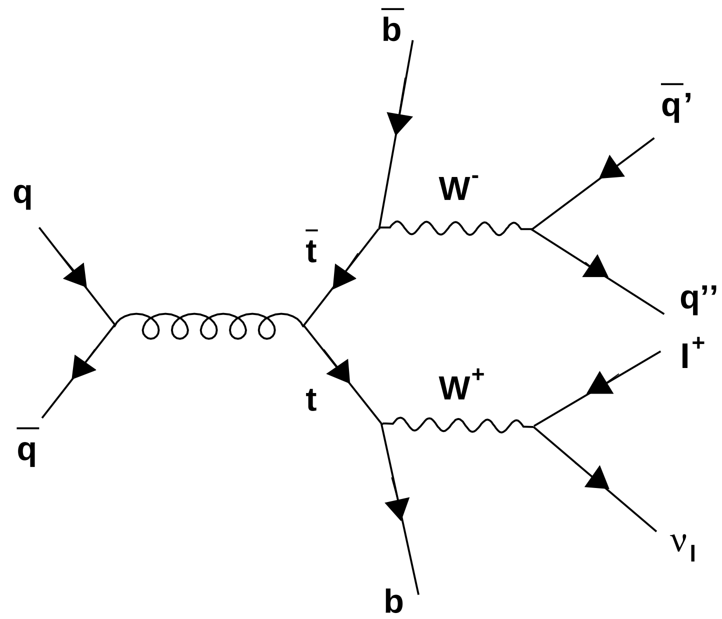
- Essential: $t \to W + b$ in almost 100% cases (weak decay within one family)
- W goes either into leptons $l\bar{\nu}_l$ or $q'q''$
- Terminology for $t\bar{t}$ decay modes based on the way W bosons decay:
  - Both Ws go into leptons: **dilepton channel**
  - One W goes on leptons, the other into quarks: **lepton+jets**
  - Both Ws go into quarks: **all–jets channel**
- What is being observed:
  - 2 energetic leptons and neutrinos, 2 b-jets
  - 1 energetic lepton and neutrino, 4 jets
  - 6 quark jets

Top decay modes (continued)

$$\Gamma_t = \frac{G_F m_t^3}{8\pi\sqrt{2}}\left(1-\frac{m_W^2}{m_t^2}\right)^2\left(1+2\frac{m_W^2}{m_t^2}\right)\left[1-\frac{2\alpha_s}{3\pi}\left(\frac{2\pi^2}{3}-\frac{5}{2}\right)\right]$$

(I have verified without QCD corrections:-)

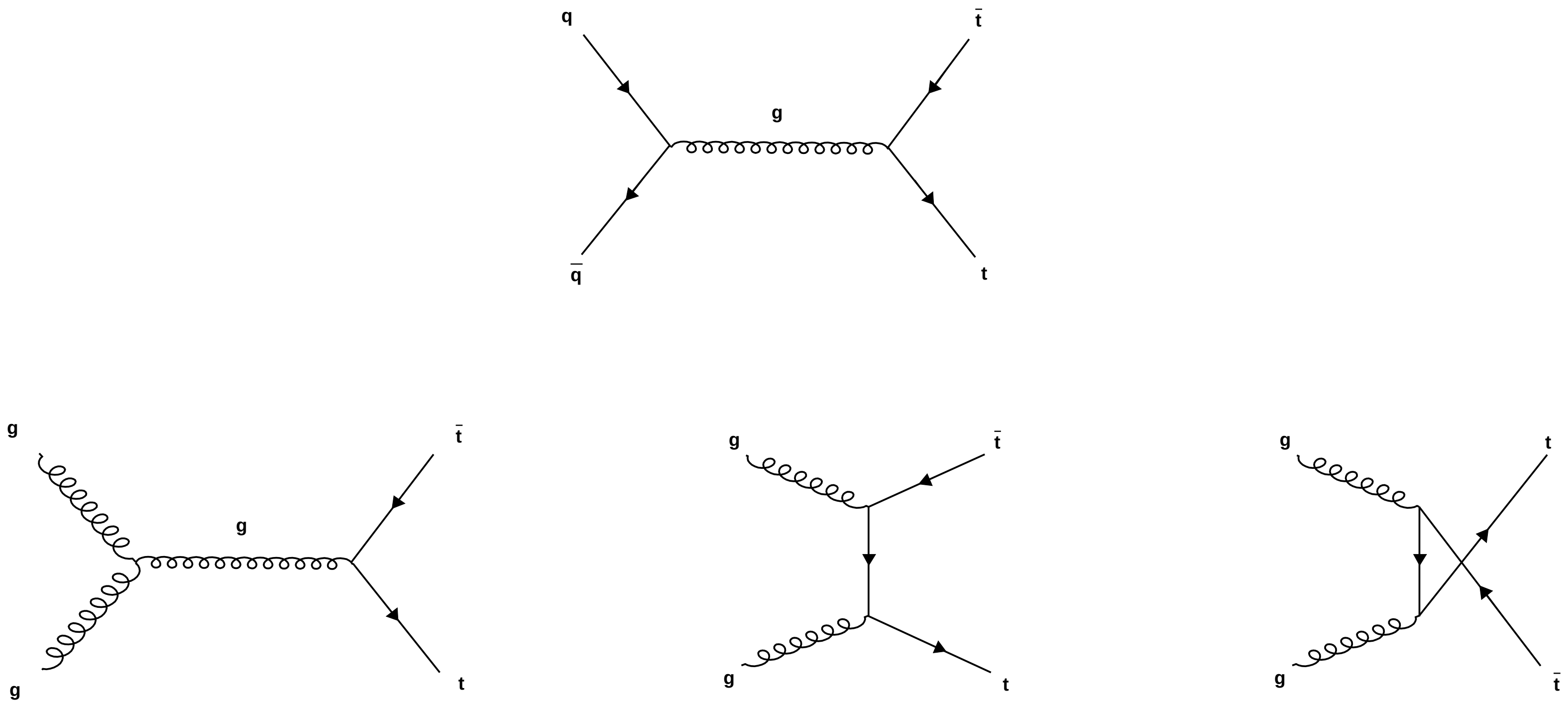




Discovery

- Announced in 1995 by CDF and DØ collaborations
- Accelerator: Tevatron – $p\bar{p}$ collider with $\sqrt{s} = 1.8\,\mathrm{TeV}$, Fermilab, Batavia (near Chicago), Illinois, USA
- Integrated luminosity of Run I period (1992–1996): $\approx 110\mathrm{pb}^{-1}$
- $p\bar{p} \to t\bar{t}$ production cross section: $\approx 6\mathrm{pb}$
- ... Therefore about 600 events expected – like searching a needle in a haystack!
- Clear signature in lepton channel (hard lepton, missing energy from neutrino, 2 b-jets)
- Numbers in Particle Data Group are CDF and DØ combined results :-)

On–shell $t\bar{t}$ production – diagrams



On–shell $t\bar{t}$ production (continued)

- $q\bar{q} \to t\bar{t}$: Only the s–channel contribution:

$$\frac{\mathrm{d}\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}\cos\theta^*} = \frac{\pi\alpha_s^2}{9\hat{s}^2}\sqrt{1-\frac{4M^2}{\hat{s}}}\left[(\hat{s}+4M^2)+(\hat{s}-4M^2)\cos^2\theta^*\right]$$

$$\hat{\sigma}_{q\bar{q}\to t\bar{t}}(\hat{s}) = \frac{8\pi}{27}\frac{\alpha_s^2}{\hat{s}^2}\sqrt{1-\frac{4M^2}{\hat{s}}}\,(\hat{s}+2M^2) \quad - \text{verified} \;\; :-)$$

- $gg \to t\bar{t}$: We have the diagonal $\hat{s}$, $\hat{t}$, $\hat{u}$ channels contributions as well as three interference terms (with negative signs!), schematically:

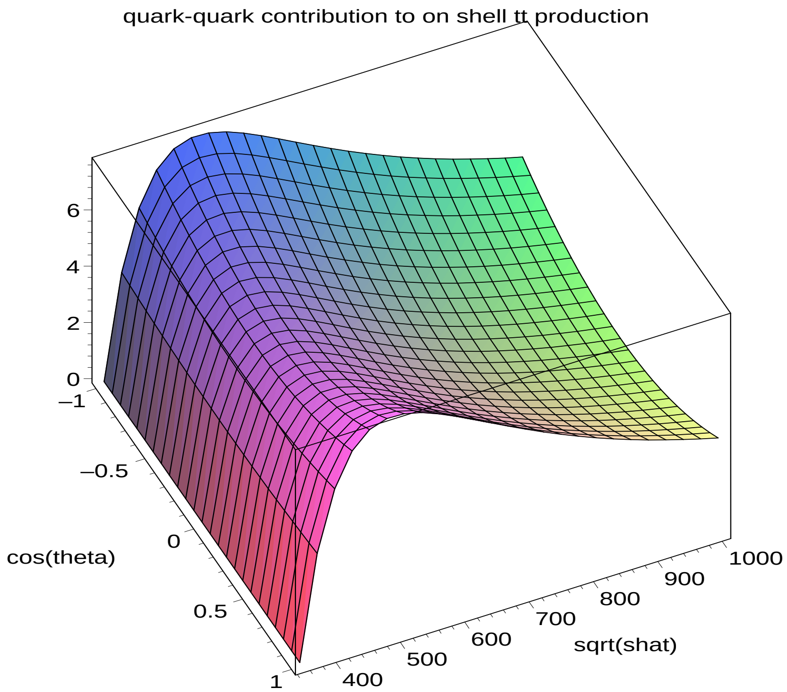
$$\overline{|\mathcal{M}_{fi}|}^2 = \overline{|\mathcal{M}_{fi}^{ss}|}^2 + \overline{|\mathcal{M}_{fi}^{tt}|}^2 + \overline{|\mathcal{M}_{fi}^{uu}|}^2 + \overline{|\mathcal{M}_{fi}^{tu}|}^2 + \overline{|\mathcal{M}_{fi}^{st}|}^2 + \overline{|\mathcal{M}_{fi}^{su}|}^2$$

$$\hat{\sigma}_{gg\to t\bar{t}}(\hat{s}) = \frac{\pi}{3}\frac{\alpha_s^2}{\hat{s}}\left[-\left(7+\frac{31M^2}{\hat{s}}\right)\frac{1}{4}\chi + \left(1+\frac{4M^2}{\hat{s}}+\frac{M^4}{\hat{s}^2}\right)\ln\frac{1+\chi}{1-\chi}\right] \quad \chi \equiv \sqrt{1-\frac{4M^2}{\hat{s}}}$$

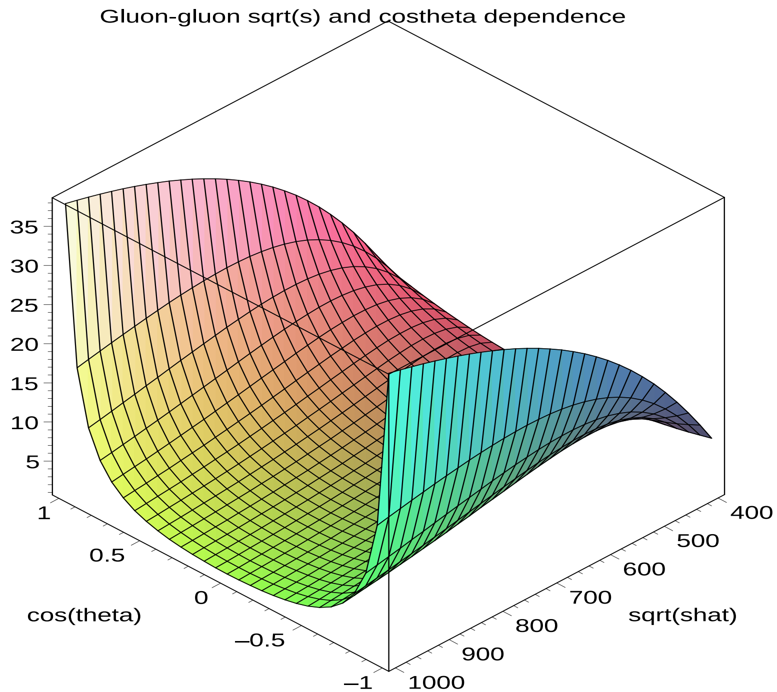
(taken from B.L.Combridge, Nuclear Physics B **151** (1979) 429)

On–shell $t\bar{t}$ production (continued)

- Processes $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ have different angular dependence!



$q\bar{q} \to t\bar{t}$



$gg \to t\bar{t}$

The off–shell process $q\bar{q} \to t\bar{t}$

- However, in reality $t\bar{t}$ are off–shell, and in order to model the situation, we have to consider different top masses in the final state.
- This yields the cross section (my result; with a proper limit for $m_1 = m_2$)

$$\frac{\mathrm{d}\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}\cos\theta^*}(\hat{s}) = \frac{\pi\alpha_s^2}{9\hat{s}^4}\,\lambda^{1/2}\,[\hat{s}^2 + 4\hat{s}m_1m_2 - (m_1^2 - m_2^2)^2 + \lambda\cos^2\theta^*]$$

$$\hat{\sigma}_{q\bar{q}\to t\bar{t}}(\hat{s}) = \frac{8\pi}{27}\frac{\alpha_s^2}{\hat{s}^4}\,\lambda^{1/2}\left[\hat{s}^2 + 2\hat{s}m_1m_2 - \frac{(m_1^2 - m_2^2)^2}{2} - \frac{\hat{s}}{2}(m_1 - m_2)^2\right]$$

where

$$\lambda \equiv \lambda(\hat{s}, m_1, m_2) = \hat{s}^2 + m_1^4 + m_2^4 - 2\hat{s}m_1^2 - 2\hat{s}m_2^2 - 2m_1^2m_2^2$$
$$= [\hat{s} - (m_1 - m_2)^2][\hat{s} - (m_1 + m_2)^2]$$

Parton model

- We are working in the limit of massless protons and quarks
- Assign proton or antiproton fourmomenta as $P_{1,2}$
- Assume that i-th parton entering the collisions carries x_i-th part of nucleon's fourmomentum:

$$p_i = x_i P_i$$

- Then the $\hat{s}$ invariant of the $q\bar{q}$ system is

$$\hat{s} \equiv (p_1 + p_2)^2 \doteq 2\, x_1 x_2\, P_1.P_2$$

$$\hat{s} = s\, x_1 x_2$$

Parton distribution functions (PDF)

- The probability that a parton q carries the fraction of nucleon's fourmomentum from $(x, x + \mathrm{d}x)$ is given by $f_q^p(x)\mathrm{d}x$.
- In other words, probability density functions for x_1 and x_2 are just $f_q^p(x_1)$ and $f_{q'}^{\bar{p}}(x_2)$
- $\hat{s}$ is product of x_1 and x_2 and its probability density can be found to be:

$$G_{qq'}(\hat{s}) = \int\limits_{\frac{\hat{s}}{s}}^{1} f_q^p \left(\frac{\hat{s}}{xs} \right) f_{q'}^p(x) \frac{\mathrm{d}x}{sx}$$

- Then the $p\bar{p} \to t\bar{t}$ cross section may be expressed as

$$\sigma_{p\bar{p}\to t\bar{t}}(s) = \sum_{q,q'} \int\limits_{\hat{s}_{thr}}^{s} G_{qq'}(\hat{s})\, \hat{\sigma}_{q\bar{q}\to t\bar{t}}(\hat{s})\, \mathrm{d}\hat{s}$$

- $G_{qq'}(\hat{s})$ simply tells us the probability that the two considered partons "meet" with such x_1 and x_2 that their CMS invariant is $\hat{s}$.

Tools for using PDF

- I used the CTEQ6 set of PDFs extracted in the leading order with $\alpha_S = 0.118$ and Gaussian numerical method taken from CERNLib
- Problem: PDFs depend on the *factorization scale* μ_F, which roughly tells the virtuality (mass) of otherwise massless partons, which are allowed to enter the process.
- Another scale μ_R comes from the renormalization procedure and appears in the cross section on the parton level. As I use LO cross sections, I got rid of this easily (no μ_R needed:)
- Good news: if summed over **all** orders of perturbation theory, $\sigma_{p\bar{p}\to t\bar{t}}$ **doesn't depend on scales** (exactly).
- Bad news: One **never** sums over all orders, so our predictions **depend on the choice of scales!**

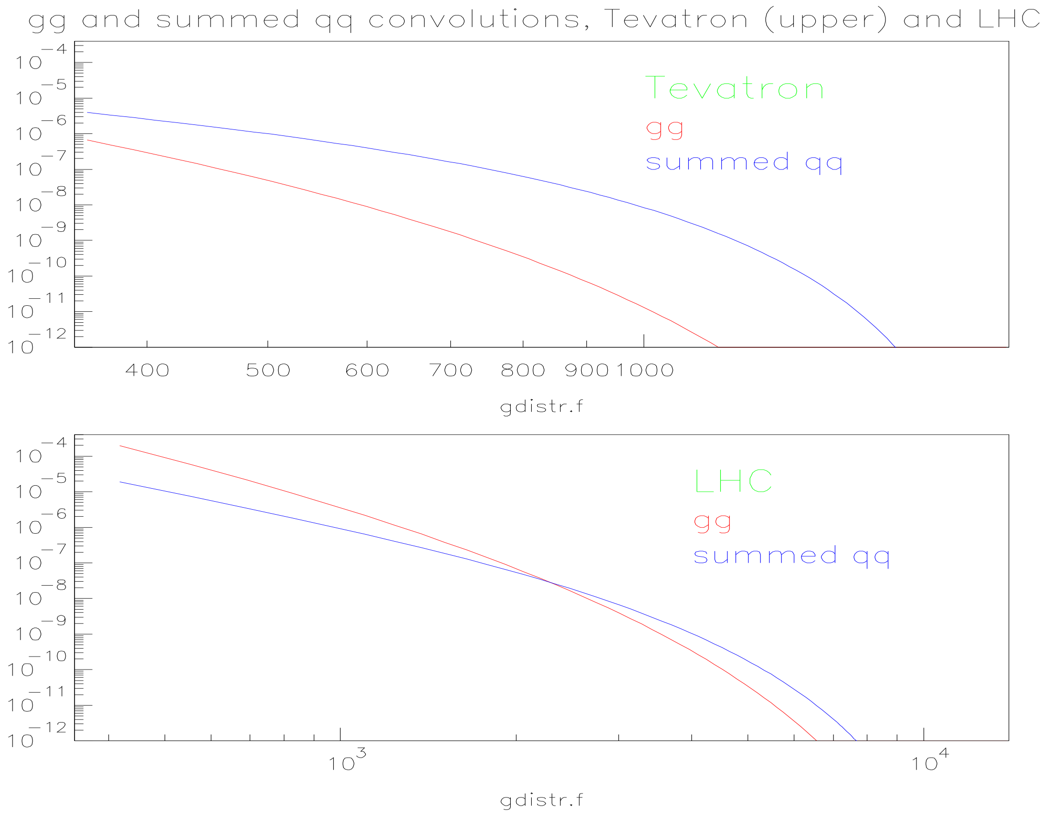
Parton combinations

- Possible parton combinations for Tevatron $p\bar{p}$ and near–future (hopefully:-) LHC pp collisions:

	Tevatron	**LHC**	
p	$\bar{p}$	p	p
g	g	g	g
u	$\bar{u}$	u	$\bar{u}_{sea}$
d	$\bar{d}$	d	$\bar{d}_{sea}$
$\bar{u}_{sea}$	u_{sea}		
$\bar{d}_{sea}$	d_{sea}		
s_{sea}	$\bar{s}_{sea}$	s_{sea}	$\bar{s}_{sea}$
c_{sea}	$\bar{c}_{sea}$	c_{sea}	$\bar{c}_{sea}$
b_{sea}	$\bar{b}_{sea}$	b_{sea}	$\bar{b}_{sea}$

Some checks of numerical integration, plotting $G_{qq'}$

- $G_{qq'}$ for gluon–gluon and summed quark–quark processes, Tevatron and LHC



Another important check: $p\bar{p} \to t\bar{t}$ cross section

- Cross sections in pb for $t\bar{t}$ production in gluon–gluon and quark–quark channels for $p\bar{p}$ or pp collisions, results taken from R. Bonciani et al., Nucl. Phys. B **529** (1998) 424 were obtained for m_t=175 GeV and $\mu_R = \mu_F$.

$\mu_R = \mu_F$	$\frac{m_t}{2}$	m_t	$2m_t$
1.8 TeV, $p\bar{p} \to t\bar{t}$, NLO theory	5.17	4.87	4.31
1.8 TeV, $p\bar{p} \to t\bar{t}$, NLO+NLL theory	5.19	5.06	4.70
2 TeV, $p\bar{p} \to t\bar{t}$, NLO theory	7.10	**6.70**	5.96
2 TeV, $p\bar{p} \to t\bar{t}$, NLO+NLL theory	7.12	6.97	6.50
14 TeV, $pp \to t\bar{t}$, NLO theory	893	803	714
14 TeV, $pp \to t\bar{t}$, NLO+NLL theory	885	**833**	794

Another important check: $p\bar{p} \to t\bar{t}$ cross section (continued)

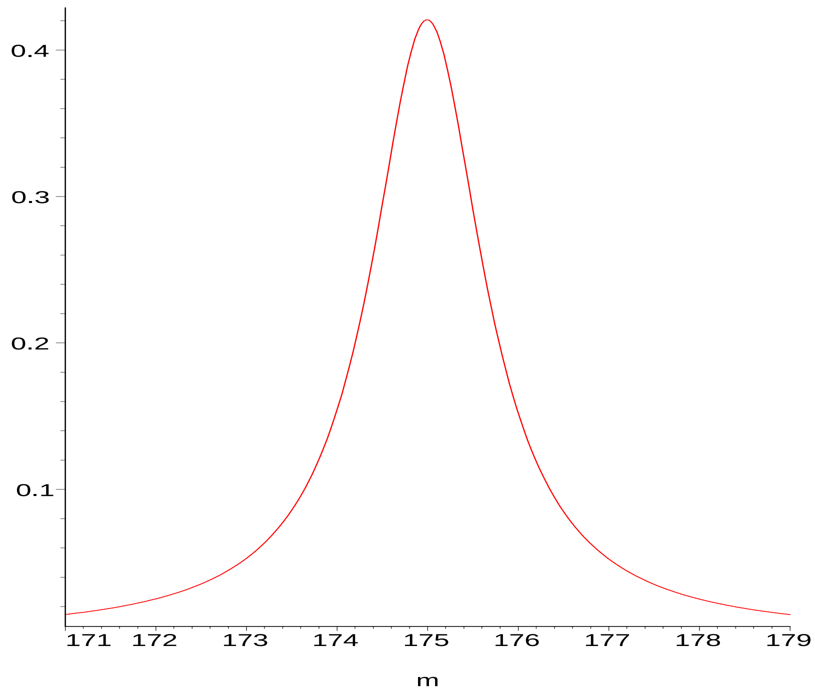
- My results: cross sections in pb for $t\bar{t}$ production in gluon–gluon and quark–quark channels for $p\bar{p}$ or pp collisions, m_t=175 GeV used. Calculated in leading order (LO) in α_S for different factorization scales μ_F

μ_F	$\frac{m_t}{2}$	m_t	$2m_t$	$\sqrt{\hat{s}}/2$	$\sqrt{\hat{s}}$
Tevatron, 1960 GeV, $p\bar{p} \to t\bar{t}$, gg	0.36	0.29	0.24	0.28	0.13
Tevatron, 1960 GeV, $p\bar{p} \to t\bar{t}$, qq	6.73	6.04	5.47	5.80	3.90
LO total cross section	7.09	**6.33**	5.71	6.08	4.03
LHC, 14 TeV, $pp \to t\bar{t}$, gg	556	515	479	491	369
LHC, 14 TeV, $pp \to t\bar{t}$, qq	73.6	75.0	75.9	75.0	76.0
LO total cross section	630	**590**	555	566	445

- …good agreement in my numbers and cited ones! :-)
- …and I got a little bit more confident I don't make a serious mistake.

Breit–Wigner distribution

- . . . a beautiful function of different ugly forms – which one to choose?
- History: Lorentz shape of spectral lines, cross section shape for baryon resonances



Breit–Wigner distribution – motivation

- Summing all contributions to the fermion propagator (m_0 is the bare mass) we get the corrected ("dressed") propagator

$$iS'_F(q) = \frac{i}{\not{q} - m_0 - \Sigma(q)}$$

- For a vector boson only the transverse part is corrected and we arrive at

$$iD'_{\mu\nu}(q) = \frac{-ig_{\mu\nu}}{q^2 - m_0^2 + \Sigma(q^2)} + q_\mu q_\nu(\ldots)$$

(terms in brackets won't contribute after contracted with fermion *vector* currents)

Breit–Wigner distribution – motivation (continued)

- Define the physical mass m so that

$$m_0^2 \equiv m^2 + \delta m^2$$

$$\mathrm{Re}\,\Sigma(m^2) = \delta m^2$$

- Use the important relation

$$\mathrm{Im}\,\Sigma(m^2) = m\Gamma(m^2)$$

- Then **around $\mathbf{q^2 \approx m^2}$** we can write

$$iD'_{\mu\nu}(q) = \frac{-ig_{\mu\nu}}{\mathbf{q^2 - m^2 + im\Gamma(m^2)}} + q_\mu q_\nu(\ldots)$$

- Squaring the denominator we get the typical Breit–Wigner shape!

Breit–Wigner distribution, my choice

- At LEP the Z and W mass analyses were performed using:

$$\rho_1(s, m_W) \equiv \frac{1}{\pi} \frac{s}{(s - m_W^2)^2 + m_W^2\,\Gamma_W^2(s)} \quad \text{with} \quad \Gamma(s) \equiv \frac{s}{m_W^2} \Gamma(m_W^2)$$

- Another proposed parametrisation was

$$\rho_2(s, m_W) \equiv \frac{1}{\pi} \frac{m_W \Gamma_W}{(s - m_W^2)^2 + m_W^2\,\Gamma_W^2} \quad \text{with constant } \Gamma_W$$

which is normalized

$$\int_0^\infty \rho_2(s) \mathrm{d}s = \frac{1}{2} + \frac{1}{\pi} \arctan \frac{m_W}{\Gamma_W} \to 1 \quad \text{for } \Gamma_W \ll m_W$$

- **Therefore, I use the form**

$$\rho(\mathbf{m_i}, \mathbf{m_t}) \equiv \frac{\mathbf{2}}{\pi} \frac{\mathbf{m_i m_t \Gamma_t}}{(\mathbf{m_i} - \mathbf{m_t^2})^\mathbf{2} + \mathbf{m_t^2}\,\mathbf{\Gamma_t^2}(\mathbf{m_i^2})}$$

Breit–Wigner distribution (continued)

- Approaches how to approximate $\Gamma(q^2)$:
  - Take simply $\Gamma(m^2)$
  - Introduce the so–called running width as $\Gamma(q^2) \equiv \frac{\sqrt{q^2}}{m}\Gamma(m^2)$
  - or define the **running width** as $\Gamma(q^2) \equiv \frac{q^2}{m^2}\Gamma(m^2)$
  - I also tried to take really $\Gamma(q^2)$ (but problems at thresholds...)

Modelling off–shell top quarks

- Let us study the by–hand modified cross section

$$\frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}(\hat{s})}{\mathrm{d}m_1\mathrm{d}m_2} \equiv \hat{\sigma}_{q\bar{q}\to t\bar{t}}(\hat{s}; m_1, m_2)\, \rho(m_1, m_t)\, \rho(m_2, m_t)$$

  (i.e. the original cross section multiplied by two B.–W. for each top)

- In data analysis, **people often require masses of both top quarks to be the same** ($m_1 = m_2$), decreasing number of unknowns in reconstruction

- To model the observed spectrum we have to **integrate over the variable $\mathbf{m_1 - m_2}$**.

- **Transformation of variables: $\mathbf{m_\pm \equiv \frac{1}{2}(m_1 \pm m_2)}$**

Integrating over m_-

- m_+ if the average mass within the $t\bar{t}$ pair!
- **Step 1:** Define the integrated cross section **on the parton level**:

$$\frac{\mathrm{d}\hat{\boldsymbol{\sigma}}_{\mathbf{q\bar{q}\to t\bar{t}}}(\hat{\mathbf{s}})}{\mathrm{d}\mathbf{m}_+} = \int\limits_{-m_+}^{m_+} \frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}m_1\mathrm{d}m_2}\,\mathrm{d}m_-$$

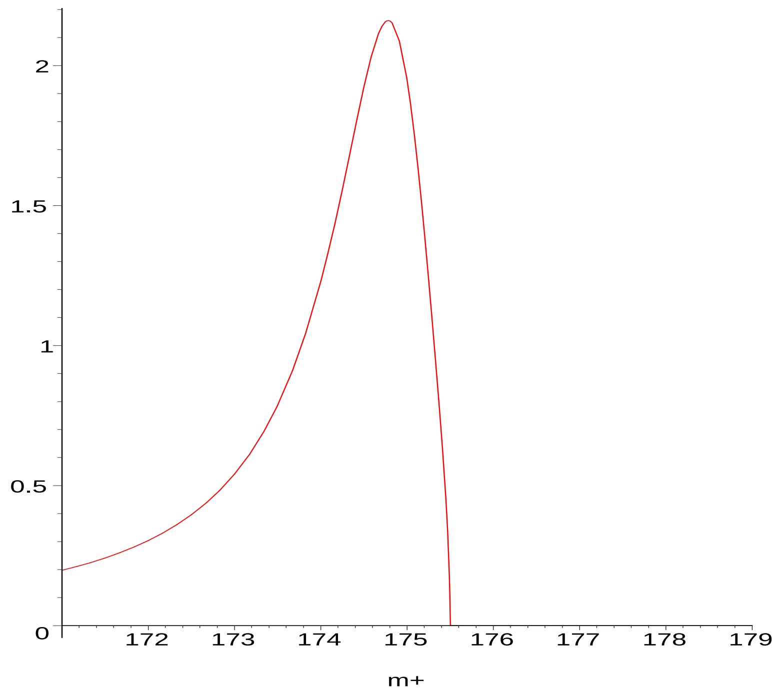
- **Step 2:** Define the integrated cross section **on the hadron level**:

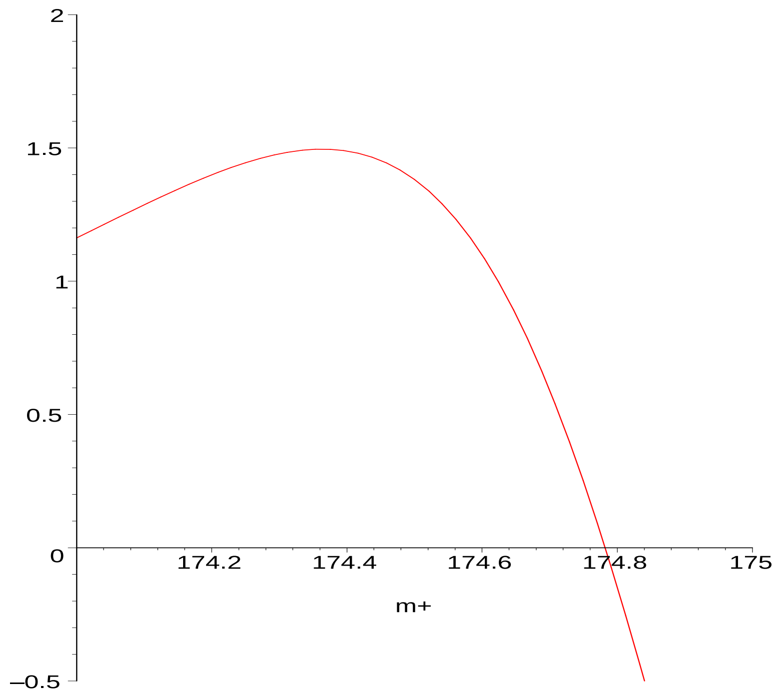
$$\frac{\mathrm{d}\boldsymbol{\sigma}_{\mathbf{q\bar{q}\to t\bar{t}}}(\mathbf{s})}{\mathrm{d}\mathbf{m}_+} = \int\limits_{4m_+}^{s} \int\limits_{-m_+}^{m_+} G_{q\bar{q}}(\hat{s})\frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}m_1\mathrm{d}m_2}\,\mathrm{d}m_-\mathrm{d}\hat{s}$$

- Still we have on-shell W bosons with $m_W = 80.4\,\mathrm{GeV}$!

Motivation – after integrating over m_-...

- Approaching the pole in BW with m_+ we get a shift of the peak!

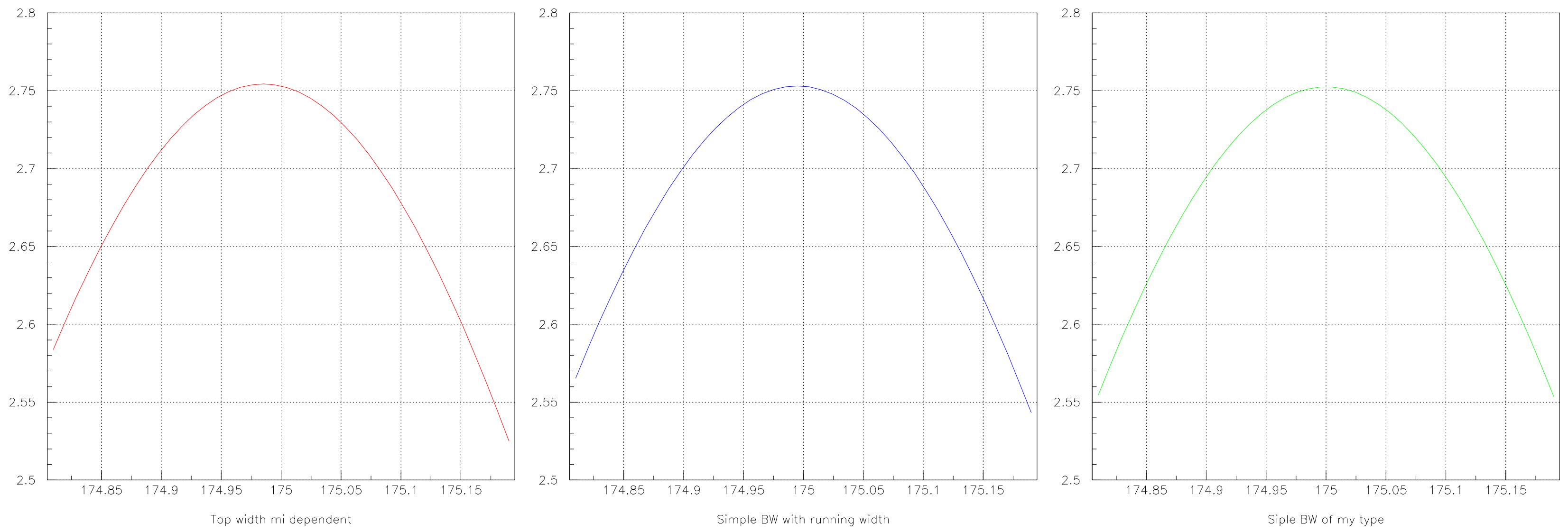




$\frac{\mathrm{d}\hat{\sigma}_{q\bar{q}\to t\bar{t}}(\hat{s})}{\mathrm{d}m_+}$ for $\sqrt{\hat{s}} = 351\,\mathrm{GeV}$ and $m_t = 175\,\mathrm{GeV}$ derivative of the plot

- Shift of the peak by $0.2\,\mathrm{GeV}$ (not much, isn't it?:-)

$\frac{\mathrm{d}\sigma_{q\bar{q}\to t\bar{t}}(s{=}1960\mathrm{GeV})}{\mathrm{d}m_+}$ for different choices of $\Gamma(m_i^2)$



$\Gamma(m_i^2)$	$\Gamma(m_i^2) = \frac{m_i^2}{m_t^2}\Gamma(m_t^2)$	$\Gamma(m_i^2) = \Gamma(m_t^2)$
Shift: $\approx$20 MeV	Shift: $\approx$10 MeV	Shift: $\lesssim$5 MeV

Trying to include off–shell W bosons – The Fivefold Way:-)

- Now let us introduce B.–W. distributions for top quarks **and** W bosons
$$\frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}m_1\mathrm{d}m_2}\,\rho(M_1,m_W)\;\rho(M_2,m_W)$$
and weight (average) over W bosons' masses.

- **Step 3:**
$$\frac{\mathrm{d}\overline{\hat{\boldsymbol{\sigma}}_{\mathbf{q\bar{q}\to t\bar{t}}}}(\hat{\mathbf{s}})}{\mathrm{d}\mathbf{m}_+} = \int\limits_{-m_+}^{m_+}\int\limits_0^{m_1}\int\limits_0^{m_2}\frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}m_1\mathrm{d}m_2}\rho(M_1,m_W)\rho(M_2,m_W)\,\mathrm{d}M_2\,\mathrm{d}M_1\mathrm{d}m_-$$

- **Step 4:**
$$\frac{\mathrm{d}\overline{\boldsymbol{\sigma}_{\mathbf{q\bar{q}\to t\bar{t}}}}(\mathbf{s})}{\mathrm{d}\mathbf{m}_+} = \int\limits_{4m_+}^{s}\int\limits_{-m_+}^{m_+}\int\limits_0^{m_1}\int\limits_0^{m_2}G_{q\bar{q}}(\hat{s})\frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}\to t\bar{t}}}{\mathrm{d}m_1\mathrm{d}m_2}\rho(M_1,m_W)\rho(M_2,m_W)\,\mathrm{d}M_2\,\mathrm{d}M_1\mathrm{d}m_-\mathrm{d}\hat{s}$$

- …so we end up with **five–fold integration** (5^{th} one is hidden in $G_{q\bar{q}}(\hat{s})$:-)

<u>Some details ...</u>

- OK, I don't expect you've read previous integrals:)
- However, one comment should take place here:
- I also had to introduce some factors to ensure energy conservation (we need $m_i \geq M_i$), possible choices are:
  - Simple cut–of: $\Theta(m_1 - M_1)\Theta(m_2 - M_2)$
  - Include two body phase space factors (I assume $m_b = 0$)

$$\frac{\lambda^{1/2}(m_1^2, M_1^2, 0)}{m_1^2} \frac{\lambda^{1/2}(m_1^2, M_1^2, 0)}{m_2^2}$$

- Similar expressions must be introduced into Step 1 and 2, where I used Θ-functions and fixed m_W.

Post Scriptum

- Preliminary results after integration over W bosons' masses:
  - No shift of the peak observed for the simplest form of BW distribution (with constant width) and Θ functions as energy cut–off.
  - No shift seen when phase space factors included
  - Computation takes about 5 hours, more results to come soon :)
- Future plans:
  - Estimate errors of the numerical integration
  - Possibly repeat the procedure for $gg \to t\bar{t}$ and LHC energy (evaluate six contributions with different top masses in the final state, integrate over angles . . .)

Acknowledgement

- Simply to all...

Acknowledgement

- OK, that would be dishonest...

Acknowledgement

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